MATHEMATICS

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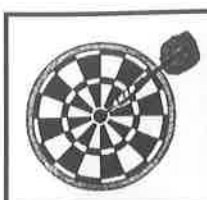

AIM POINTS
MATHEMATICS
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XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XII (PQRS)

MEAN AND VARIANCE OF A RANDOM VARIABLE

& Their Properties

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THINGS TO REMEMBER

1. Let S be the sample space assoiated with a given random experiment. Then, a real valued function X which assigns to each event W $\in$ S to a unique real number X (w) is called a random bariable.

   In other words, a random variable is a real valued funciton having domain asthe sample space associated with a random experiment.

2. If a random variable X takes values x_1, x_2 , ..., x_n with respective probabilities p_1, p_2, ...,p_n, then

X :	x_1	x_2	.....	x_n
P(X) :	p_1	p_2	.....	p_n

   is known as the probability distribution of x.

3. The probability distribution of a random variable X is defined only when we have the various values of the random variable e.g. xx_1, x_2 , ..., x_n together with respective

   probabilities p_1, p_2, ...,p_n, satisfying $P(X=x)\begin{cases} 0.1 & ,\text{if } x=0 \\ kx & ,\text{if } x=1 \text{ or } 2 \\ d(5-x) & ,\text{if } x=3 \text{ or } 4 \\ 0 & ,\text{otherwise} \end{cases}$

4. If X is a discrete random variable which assumes balues x_1, x_2 ,x_3, ..., x_n with respective probabilities p_1, p_2, p_3, ...,p_n, then the mean $\bar{X}$ ofX is defined as

   $\bar{X}$ $p_1 x_1 + p_2 x_2 + ... + p_n x_n$ or, $\bar{X} = \sum_{i=1}^{n} Pi\, xi$

   The mean of a random variable X is also known as its mathematical expectation or expected value and is denoted by E (X).

EXERCISE – 1

1. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the value x has the following form, where k is some unknown contant.

$$p(X=x)\begin{cases} 0.1 & ,\text{if } x=0 \\ kx & ,\text{if } x=1 \text{ or } 2 \\ d(5-x) & ,\text{if } x=3 \text{ or } 4 \\ 0 & ,\text{otherwise} \end{cases}$$

   (i) Find the blue of k ?

   (ii) What is the probability that you study at least two hours ?

   (iii) Exactly towo hours ?

   (iv) At most towo hours ?

2. A random variable X has the following probability distribution values of X,

X :	0	1	2	3	4	5	6	7
P(X) :	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

   Find each of following :

   (a) k (b) P(X < 6) (c) P(X ≤ 6) (d) P(0 < X < 5)

3. A random variable X can take all non-negative integral values and the probability that X takes the value r is proportional to α^r $(o < \alpha < 1)$. Find $P(X = 0)$.
4. Find the probability distribution of X, the number of heads in two tossed of a α in (or a simultaneous toss of thwo coins).
5. Three cards are drawn from a pack of 52 playing cards. Find the probability distribution of the number of aces.
6. Four bad oranges are mixed accidently with 16 good oranges. Find the probability distribution of the member of bad oranges in a draw of thwo oranges.
7. A coin is tossed until a head appears of the tail appears 4 times in succession. Find the probability distribution of the number of tosses.
8. An urn contains 4 white and 3 red balls. Find th probability distribution of the number of red balls in a random drqwn of three balls.
9. Two cards are drawn without repiacement from a well-shuffled deck of 52 cards. Determine the probability distribuiton of the number of face cards (i.e. Jack, Queen, King nad Ace).
10. A coin is baised so that he head is 3 tiems as likely to occur as tail. If coin is tossed twice. find the probability distribution for the number of tail.
11. Let X be arandom variable which assumes vlues x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$. Find the probability distribution of X.
12. A random variable X tkes the values 0, 1, 2 and 3 such that :
    $P(X = 0) = P(X > 0) = P(X < 0)$; $P(X = -3) = P(X = -2) = P(X = -1)$
    $P(X = 1) = P(X = 2) = P(X = 3)$. Obtain the probability distribution of X.
13. A bag contains 4 red nad 6 black balls. Therr balls are drawn at random. Find the probability distribution os the umber of red balls.
14. A class has 15 students whose ages are 14, 17, 15, 21, 19, 20, 16, 18, 17, 20, 17, 16, 19 and 20 years respectively. One student is selected in such a manner that each has the same chance of being selected and the age X of the selected sudent is recorded. What is the probability distribution of the random variable X ?
15. Five defective bolts are accidently mixed with twenty good ones. If four bolts are drawn at random from this lot, find the probability distribution of the number of defective bolts.
16. Two cards are drawn successibely without replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number os aces.
17. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.
18. Two cards are drawn simultanously from a well-shuffled dack of 52 cards. Find the probability distribution of the number os successes, when getting a spade is considered 4 success.
19. A fair die is tossed twice. If the number appearing on the top is less than 3, it is a success. Find the probability distribution of number of successes.
20. An urn contains 5 red and 2 black balls. Two balls are randomly selected. Let X represent the number of black balls. What are the possible values of X. Is X a random variable ?
21. From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

22. If a pair of dice is thrown and X demotes the sum of the numbers on them. Find the probability distribution of X. Also, find the expectation of X.
23. Find the mean, variance and standard deviation of the number of heads in a simultaneous toss of three coins.
24. Two dice are thrown simultaneously. If X denotes the number of sexes, find the expectation of X.
25. Two umbers are selected at random (with replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E (X) and Var (X).
26. In a metting 70% of the members fovour a certcin proposal, 30% being opposed. A member is selected at random and let X = 0 if the opposed, and X = 1 if he is in favour. Fin E (X) and Var (X).
27. Find the probability distribution of the number os successes in two tossed of a die, where a success is defined as 'getting a number greater than 4'. Also, find the mean and variance of the distribution.
28. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the items in the sample are drawn one by one without replacement, Find
    (i) The probability distribution of X.
    (ii) Mean of X
    (iii) Variance of X
29. There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the number on two cards drawn. Find the mean and variance.
30. In a game, a person is paid Rs. 5 if he gets all heads or all tails ehen three coins are tossed, and he will pay Rs. 3 if either one or two heads show. What can he expect ot win on the average per game ?
31. Let X denote the number of vowels in word selected at random from this sentence. Find the expected value and standard deviation of the random variable X. (Consider X as a word with one letter).